1. Show that the dimension of a complex semisimple algebraic group is at least 3.

2. Explain why a finite group is an algebraic group. Show that a finite group consists of semisimple elements only.

3. Calculate the group of algebraic characters of the following groups:

   (a) The cyclic group of order n.

   (b) The symmetric group S_n.

   (c) The alternating subgroup $A_n \subseteq S_n$.

   (d) The n-dimensional complex torus.

   (e) The general linear group $\mathbf{GL}_2(\mathbb{C})$.

   (f) The special linear group $\mathbf{SL}_2(\mathbb{C})$.

   (g) $P = \left\{ \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & f & g \end{bmatrix} \in \mathbf{GL}_3(\mathbb{C}) \right\}$.

   (h) $Q = \left\{ \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \in \mathbf{GL}_4(\mathbb{C}) \right\}$.

   (i) $H = \left\{ \begin{bmatrix} 1 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & g \end{bmatrix} \in \mathbf{GL}_3(\mathbb{C}) \right\}$.

4. Which of the groups in the previous exercise are simple, semisimple, or reductive?

5. Determine the Picard group of $\mathbb{P}^n$.

6. Determine the Picard group of the Grassmann variety of 2 planes in $\mathbb{C}^4$.

7. Show that $\mathbf{GL}_n(\mathbb{C})$ is not simply connected in the Hausdorff topology. Show directly that $\mathbf{GL}_n(\mathbb{C})$ is not algebraically simply connected.

8. Let G be a complex semisimple algebraic group. Determine the equivariant Picard group $\mathrm{Pic}_{G\times G}(G)$, where $G \times G$ acts on G as follows:

$$(g, h) \cdot x = gxh^{-1} \qquad (g, h, x \in G).$$