For the next four problems the following notation is fixed: let G denote $\mathbf{SL}_3(\mathbb{C})$, let B denote the Borel subgroup of upper triangular matrices in G, and let T denote the maximal diagonal torus in B.

1. Compute the weight lattice of (G, B, T).

2. Compute the Weyl group W of (G, T).

3. Let $\{\varpi_1, \varpi_2\}$ be the set of fundamental dominant weights of (G, B, T). Compute the W-orbits of ϖ_1 and $\varpi_1 + \varpi_2$.

4. Let W denote the vector space of 3×3 skew-symmetric matrices on which G acts by the congruence action:

$$g \cdot A = gAg^{\top} \qquad (g \in G, A \in W).$$

   Show that W is an irreducible representation of G. Find the highest weight of W.

For the next three problems the following notation is fixed: let G denote $\mathbf{SL}_2(\mathbb{C})$, let B denote the Borel subgroup of upper triangular matrices in G, and let T denote the maximal diagonal torus in B.

1. Find the character group $\ddot{\mathbf{O}}(B)$ of B.

2. Let V denote the two dimensional complex vector space of 2×1 column matrices on which G acts by matrix multiplication. Compute the weight lattice of V in $\ddot{\mathbf{O}}(B)$.

3. For each $n \in \mathbb{Z}^+$ find an irreducible rational representation V of G such that $\dim V = n$. Show that in each dimension, there is exactly one irreducible representations of G.