1. Consider the natural action of GL_n on the $n-1$ dimensional projective space $\mathbb{P}^{n-1}$. Let $\lambda : \mathbb{G}_m \to T$ be a 1-psg of the n-dimensional diagonal torus $T \subseteq GL_n$. What can you say about the fixed points of the action of $\lambda(\mathbb{G}_m)$ on $\mathbb{P}^{n-1}$? If $n = 2$, then how many $\lambda(\mathbb{G}_m)$-orbits are there in $\mathbb{P}^1$?

2. Prove that $\mathbb{P}^1$ is isomorphic to GL_2/B, where B is the Borel subgroup of upper triangular matrices in GL_2.

3. Prove that $GL_2/B \cong SL_2/B_0$, where $B_0 = B \cap SL_2$.

4. Let T_0 be a maximal torus in B_0, where $B_0 \subset SL_2$ is as in the previous example.

   (a) Show that SL_2/T_0 is an affine variety.

   (b) Show that SL_2/T_0 is a spherical variety.

5. Let B_0 be as in the previous example.

   (a) Compute the unipotent radical U_0 of B_0, and show that SL_2/U_0 is a spherical variety.

   (b) Show that SL_2/U_0 is a quasi-affine variety.

   (c) Explain why SL_2/U_0 is not an affine variety.

6. Show that SL_n/U, where U is the unipotent radical of a Borel subgroup of SL_n is a spherical variety.

7. Let T_0 denote the maximal diagonal torus of SL_3.

   (a) Prove that SL_3/T_0 is an affine variety.

   (b) Prove that SL_3/T_0 is not a spherical variety.